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Packing of Irregular Particles

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The packing of narrow fractions of irregular particles to a given density has been studied theoretically and experimentally. The equations given in Furnas' classical investigation have been altered to give more commonly valid expressions. It has also been found necessary to propose a new model, which better meets the experimental demands. An example is given of how to solve a multicomponent system by using this model.

1. Introduction

In several technical fields the packing of irregular particles to a given density is of great importance. Particularly this is true, when bodies of a given porosity or a high density and strength have to be manufactured. In the present paper the factors will be studied, which influence the packing of irregular particles to a minimum void.

The task is to fill with particles the space occupied by air in a packing. To do this, it is necessary to know the shape, volume and location of the air inclusions. In practice, however, it is impossible to meet these requirements, particularly as it is also necessary to place the right particle in the right inclusion.

Thus it is necessary to use simplified models. The only one so far published was developed by Furnas [1] in 1931. He tackles the problem by assuming that to a packing of a coarse component, a fine component can be introduced, which fills the space between the particles of the coarse component without expanding the total volume. The word component is used to define a certain fraction of particles. In practice, however, the total volume is not constant. In the present paper Furnas' equations have been altered to give more commonly valid expressions. Further a new model is proposed, which better meets the experimental demands.

2. Development of Furnas' Equations

It is possible to derive expressions for the weight fractions in a packing with maximum density, which are simpler and in better agreement with practice than Furnas' [1] equations.

Denoting:

d the mean value of the size of the largest and smallest particles in a fraction, i.e. mean aperture value of the two adjacent sieves;

d_n the index denoting the component. $d_1 > d_2 > d_3 \ldots$;

δ_n the apparent density of d_n;

ρ_n the particle density of d_n. The particle density includes closed pores and thus varies with particle size;

m_n the weight of d_n.

The apparent density of the coarsest component, d_1, is then δ_1 and the particle density ρ_1. By subtracting the inverse value of the particle density from the inverse value of the apparent density the volume of air per unit weight of component d_1 is given.

$$w_1 = \left(\frac{1}{\delta_1} - \frac{1}{\rho_1}\right) . \quad (1)$$

The air content of m_1 weight units of component d_1 is

$$\text{air volume of } d_1 = m_1 w_1 . \quad (2)$$

Equation 2 gives the volume available for component d_2. Using Furnas' model the weight m_2 of d_2 can be calculated:

$$m_2 = \delta_2 m_1 w_1 \quad (3)$$

giving the ratio

$$\frac{m_1}{m_2} = \frac{m_1}{m_1 \delta_2 w_1} = \frac{1}{w_1 \delta_2} . \quad (4)$$

The theoretical apparent density for a two component mixture is

$$\delta_T = \frac{m_1 + m_2}{V} = \frac{m_1 + m_1 w_1 \delta_2}{m_1/\delta_1} = (\delta_1 + \delta_1 \delta_2 w_1). \quad (5)$$

In the same way the ratio between m_2 and m_3 can be derived

$$\frac{m_2}{m_3} = \frac{1}{w_2\delta_3}\,. \tag{6}$$

Generally for the $(n+1)$ component this can be written

$$m_{n+1} = m_n w_n \delta_{n+1} \tag{7}$$

and the theoretical apparent density for n components

$$\delta_T = \delta_1 + \delta_1\delta_2 w_1 + \delta_1\delta_2\delta_3 w_1 w_2 + \delta_1\delta_2\delta_3\delta_4 w_1 w_2 w_3 + - - - + + \delta_1\delta_2 \ldots \delta_{n-1}\delta_n w_1 w_2 \ldots w_{n-2} w_{n-1}\,. \tag{8}$$

3. Experimental

The equation 7 is in a very convenient form. It also gives the solution for a n-component system. In order to check its validity an experimental series was carried out. Fired clay (chamotte) was chosen as particle material because both δ and ρ vary with particle size due to closed pores. As base component a fraction between 4.0 and 5.0 mm was chosen. This was mixed in different proportions with the following fractions: 1.0 to 1.5 mm; 0.6 to 1.0 mm; 0.2 to 0.6 mm and 0.074 to 0.2 mm.

We define

$$r = \frac{\text{mean value of coarse fraction}}{\text{mean value of fine fraction}} = \frac{d_1}{d_2}$$

In the different mixtures the following r-values will then be obtained: 3.6; 5.6; 11.3; 32.8. The components were thoroughly mixed and the loose volume of the packing was determined. Knowing the amount of material the corresponding apparent density can be calculated. This is shown by the upper curve in figs. 1, 2, 3 and 4. The limits given are the 95% probability for the average values.

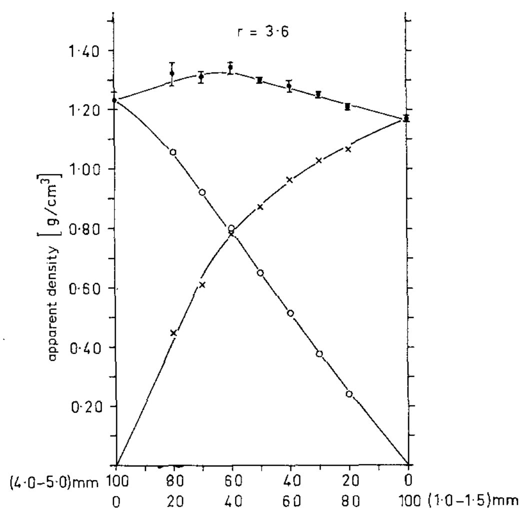

Figure 1 Experimental results for the mixture $r = 3.6$. In the figure are also shown the apparent densities of the components in the mixture.

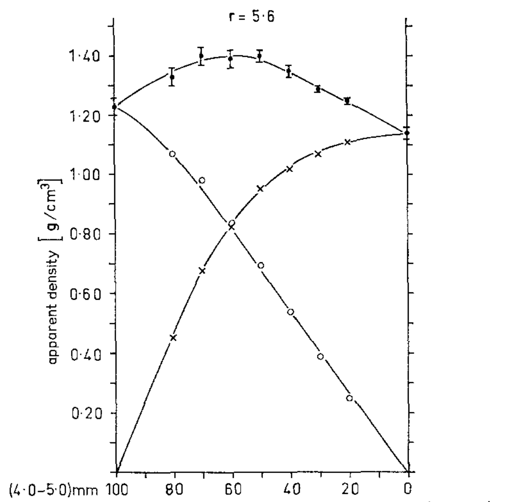

Figure 2 Experimental results for the mixture $r = 5.6$.

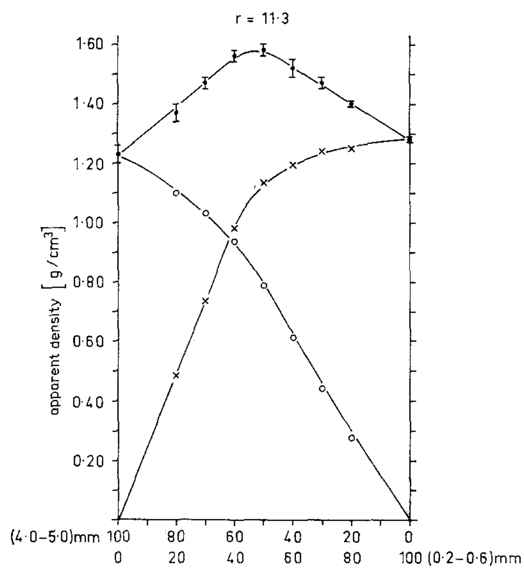

Figure 3 Experimental results for the mixture $r = 11.3$.

4. Discussion

The particle densities of the fractions were determined pycnometrically and are shown in table I.

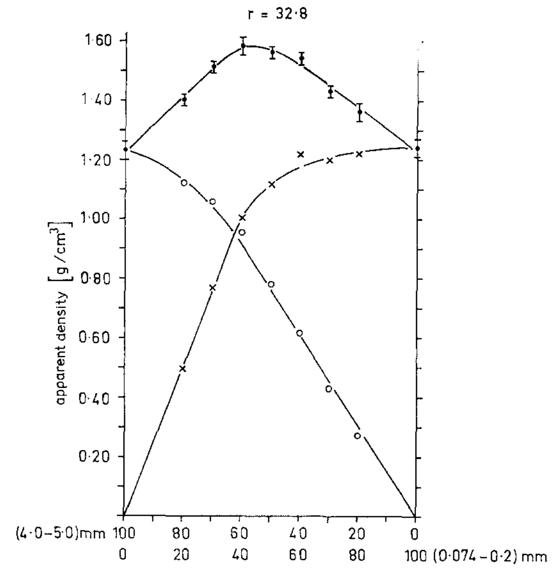

Figure 4 Experimental results for the mixture $r = 32.8$.

TABLE I Particle densities of the fired clay fractions

Fraction	$\rho_1, \rho_2, \rho_3, \rho_4, \rho_5$
4.0 to 5.0 mm	2.57 g cm^{-3}
1.0 to 1.5	2.65
0.6 to 1.0	2.63
0.2 to 0.6	2.67
0.074 to 0.2	2.72

Using equation 7 and the experimental values of δ_1 and δ_2 given at 0 and 100 in figs. 1 to 4 the mixture ratio for the densest packing can be calculated. The results are shown in table II.

TABLE II The densest mixture according to equation 7

r	3.6	5.6	11.3	32.8
m_1	66.9	67.4	64.7	65.5
m_2	33.1	32.6	35.3	34.5
	100.0%	100.0%	100.0%	100.0%

By comparing table II to figs. 1 to 4 it can be seen that the calculated mixture ratio falls within the optimum region when $r = 3.6$ and $r = 5.6$, but to the left of it when $r = 11.3$ and $r = 32.8$. Thus equation 7 obviously does not always give the correct mixtures ratios. This disagreement is due to the fact that the expansion after mixing is neglected.

5. A New Model

As the model used did not agree with the experimental results, it was decided to alter it. We assume we have a two component mixture and take away the finer component without volume contraction. The coarse particles then remain hanging in the space, as shown in fig. 5.

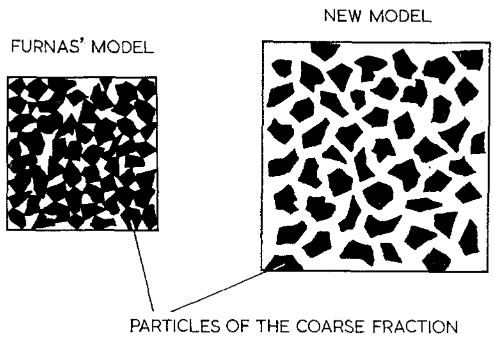

Figure 5 Comparison between Furnas' model and the proposed model.

The apparent density of the coarse component in this expanded state, is the true apparent density of the component in the mixture and should be used in equation 7. When defined in this way, the apparent density obviously varies with both the weight and the r-value.

It seems very improbable that the apparent density in the mixture can be calculated theoretically. Experimentally, however, the apparent density can easily be found, as we know the total volume of the packing and the weight of the coarse component. This is shown for the examined fractions by the open circle curves (O) in figs. 1 to 4. Knowing the particle density of the coarse component it is possible to calculate the volume of the dense material of the coarse component. By subtracting this from total volume of the packing the volume available for the fine component is obtained. This volume and the amount of fine component used then give the apparent density in the mixture for the fine component. These values are shown by the ×-curves in figs. 1 to 4. From the figures it can now be seen, that not even for very high partial values of one component does the apparent density in the mixture equal the apparent density of the pure fractions. In other words, even small additions of a fine component cause an expansion.

When the apparent density in mixtures now is known, correction factors can be worked out for calculating apparent densities in packings. The correction factor for the component d_1 is denoted by x. By multiplying the apparent density of the pure fraction with x the apparent density in a

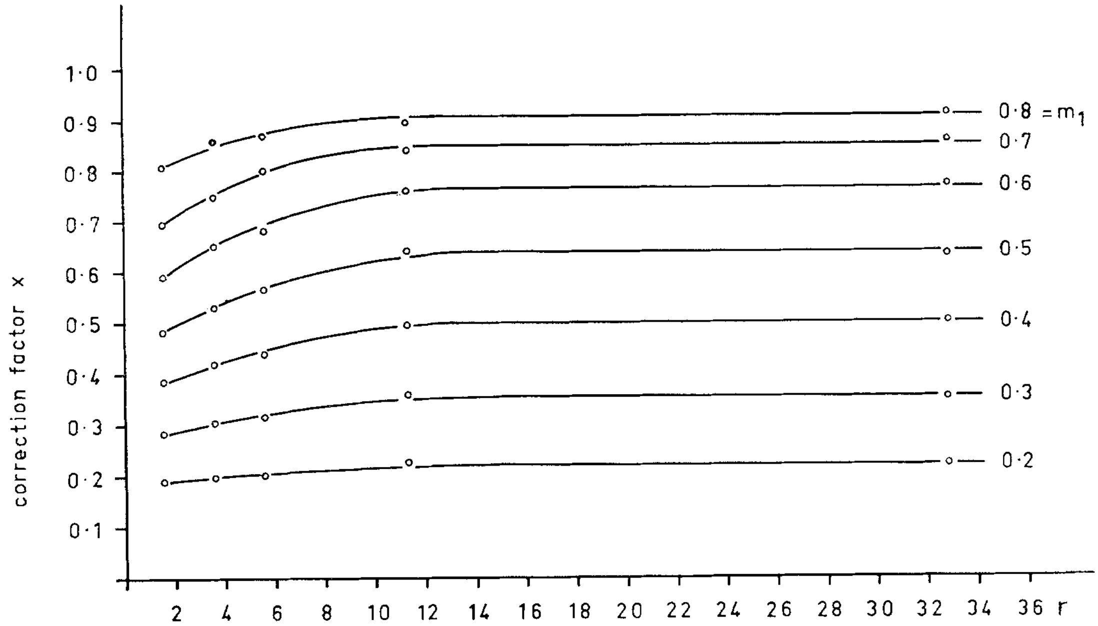

Figure 6 The correction factor x as a function of r and m_1.

given mixture is obtained. The correction factor x for the investigated particles is graphically shown in fig. 6. The correction factor for the component d_2 is denoted y and graphically shown in fig. 7.

When introducing the new model the above derived equations have to be altered. δ_1 has to be replaced by $x\delta_1$ and δ_2 by $y\delta_2$. The equation 5 then reads

$$\delta_T = x\delta_1 + x\delta_1 y\delta_2 w_{1x} \tag{9}$$

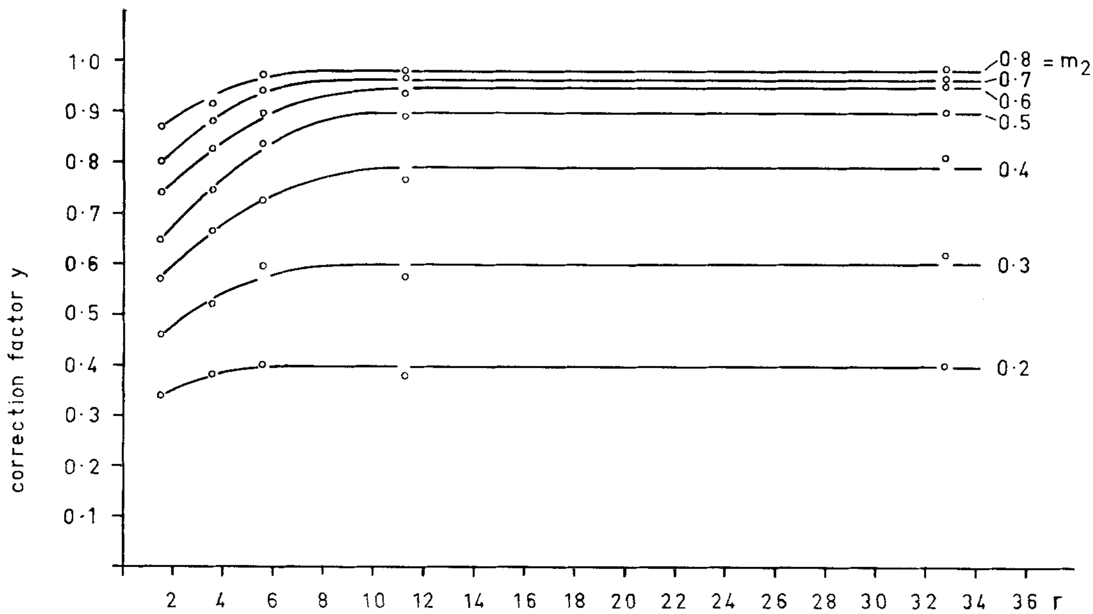

Figure 7 The correction factor y as a function of r and m_2.

where

$$w_{1x} = \left(\frac{1}{x\delta_1} - \frac{1}{\rho_1}\right)$$

or when inserting w_{1x}

$$\delta_T = x\delta_1 + y\delta_2 - \frac{xy\delta_1\delta_2}{\rho_1}. \qquad (10)$$

The densest packing is obtained using equation 10 for different compositions and choosing the densest one.

6. Application to Three-Component Systems

An attempt was also made to solve a three-component mixture. The following fractions were used: d_1: 4.0 to 5.0 mm; d_2: 0.6 to 1.0 mm; d_3: 0.074 to 0.2 mm.

The densest packing, giving an apparent density of 1.64 g cm^{-3}, was experimentally obtained for the mixture

$$\begin{array}{ll} d_1: & 40\% \\ d_2: & 30\% \\ d_3: & 30\% \\ \hline & 100\%. \end{array}$$

By using either equation 10 or figs. 2 and 4 the densest packing for the mixtures $(d_1 + d_2)$ and $(d_1 + d_3)$ is obtained when:

$$\begin{array}{lll} d_1: & 60 & 60 \\ d_2: & 40 & \\ d_3: & & 40 \\ \hline & 100 & 100. \end{array}$$

Thus we start from $(d_1 + d_2)$ mixed in the above given ratio. In order to keep the right ratio between d_1 and d_3 in the three-component mixture, the following condition must be satisfied:

$$\begin{array}{llll} d_1 & 60 & & 42.8\% \\ d_2 & 40 & \text{i.e.} & 28.6\% \\ d_3 & 40 & & 28.6\% \\ \hline & 140 & & 100.0\%. \end{array}$$

This agrees very favourably with the experimental result, as a change of 5% in the coarsest component hardly affects the apparent density of the mixture.

The experiments also show that in a multi-component mixture the apparent density is mainly dependent on the ratio between the coarsest and the finest components, the component in between having very small effect. For instance the maximum apparent density for $(d_1 + d_3)$ is 1.58 g cm^{-3} (see fig. 4). When d_2 is added the value increases to 1.64 g cm^{-3}. The apparent density of the separate components is about 1.2 g cm^{-3}. Thus it has not been found necessary in the case described above to find a certain ratio between d_2 and d_3.

Mixtures of more than three components are of less interest, as the packing density does not increase. Clews and Green [2], for instance, have studied mixtures of up to five components and found the densest packings at three.

7. Conclusions

In studying the packing of narrow fractions of irregular particles it has been found useful to alter the equations given by Furnas [1] to more commonly valid expressions. When testing these equations, the experiments proved it necessary to alter Furnas' model, as a volume expansion takes place when mixing two or several fractions. The new model proposed differs from the old essentially in that the apparent density of a component is defined as the apparent density of the component in the final mixture. An application to a three-component system gives very good agreement between calculated and experimental results.

References

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